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Mass gap in Kaluza-Klein spectrum in a network of brane worlds

Soonkeon Nam*

Department of Physics, Harvard University
Cambridge, MA 02138
E-mail: nam@pauli.harvard.edu

Abstract: We consider the Newton's force law for brane world consisting of periodic configuration of branes. We show that it supports a massless graviton. Furthermore, this massless mode is well separated from the Kaluza-Klein spectrum by a mass gap. Thus most of the problems in phenomenology coming from continuum of Kaluza-Klein modes without mass gap are potentially cured in such a model.

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*Permanent address: Department of Physics, Kyung Hee University; Seoul, 130-701, Korea, E-mail: nam@string.kyunghee.ac.kr

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There has been a considerable interest in the model where the standard model is confined to a $(3+1)$-dimensional subspace in the higher dimensions [1, 2, 3]. This is a renewal of old ideas [4, 5] in light of recent developments of string theory, especially due to the fundamental role that extended objects, i.e. branes, play a fundamental role. There has been a lot of works related to this recently, such as generalization of the RS model [6, 7, 8], in relation to supergravity or superstrings [9], in relation to cosmology [10], and also some phenomenological consequences [11].

An alternative understanding of the hierachy problem of the electroweak and gravitational mass scales is one of the major advantage of such a scenario. In this scenario, our understanding of classical and quantum gravity has to be critically reanalyzed too: Newton's law is affected by the Kaluza-Klein modes of the large extra dimensions, and gravity may become strong at a scale of few TeV in the full $(4+n)$-dimensional space. Furthermore, the effect of virtual exchange of Kaluza-Klein towers of gravitons might be detected in colliders in a forseeable future [12]. In the original formulation of Randall and Sundrum, a continuous spectrum of Kaluza-Klein modes of graviton arises without any mass gap [3]. However, to have a well-defined effective field theory, it is desirable to have a solution where the graviton is well separated by a mass gap from the Kaluza-Klein modes. In this paper, we consider a model with the desired spectrum of Kaluza-Klein with a mass gap without direct reference to supergravity. This is when we have a periodic array of thin branes, in light of recent proposals for a network such braneworlds [13], or a folded brane [14], producing many idential braneworlds [15]. Such a picture raises an interesting question. What will be the effect on our world from the presence of other worlds? Some indirect effects such as messengers of supersymmetry breaking [13, 14], or cosmological/astrophysical effects were discussed [14]. Here we will discuss one another aspect of such a network of braneworld, which might be more sensitive to the shape of the network of the braneworlds. It is the spectrum of Kaluza-Klein modes, which might perhaps be explorable at LC and Muon Colliders [12]. As is quite familiar from condensed matter physics, which deals with periodic systems (one or higher dimensional) in many cases, the energy levels of have a distinct feature of the periodicity. This is the occurance of forbidden zones and allowed bands in energy spectrum [16]. If we recall that the fluctuation equations of modes around a stable BPS configurations satisfy (supersymmetric) quantum mechanics [17], and the spectrum of Kaluza-Klein spectrum comes from the energy spectrum of such a Schrödinger equation with the potential determined by the shape of the stable BPS object, we expect that a periodic configuration of such objects will lead to Schrödinger equation with a periodic potential and the Kaluza-Klein spectrum will have mass gaps, reflecting the *band* structure. One might argue that there will be no distiction between this and the torus compactification around a circle. However, since it is likely that the number of large extra dimensions will be equal or larger than 2, there are possibilities of highly non-trivial network of braneworlds, as we have witnessed from the many different ways that nanotubes can form and affect the electronic structure [18].

Here we will be mainly concerned with the Kaluza-Klein modes of the graviton. This is because the original formulation of Randall and Sundrum necessarily has a continuum of Kaluza-Klein modes without any mass gap, and the very low lying modes has been a discomforting factor of the model. Here we will explicitly calculate the mass gap arising from a periodic system of 3 branes. There has been some works [19] having mass gaps from a distribution of D-branes in the context of five-dimensional supergravity, however the spectrum is different from what we consider here.

Although we have confined our calculations to a simple one-dimensional case this method can be easily generalized to higher-dimensional cases as well as more complicated networks, as well as other potentials for the fluctuation equations from smooth models [20, 21]. For example, for the graphite-like structure (or nanotube-like structures) we can utilize the hexagonal symmetry of the system and obtain the band structure, although we might have to resort to numerical methods eventually. Let us now consider the solution five-dimensional metric that respects four-dimensional Poincaré invariance [3], plus the linearlized tensor fluctuations $h_{\mu\nu}$ around it.

$$ds^2 = \left(e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x,y)\right) dx^\mu \, dx^\nu + dy^2 \,. \tag{1}$$

The spectrum $h_{\mu\nu}$ satisfies the Schrödinger equation, with the appropriate change of variables [3]: $h(x,y) = \psi(y)e^{ip\cdot x}$, $\hat{\psi}(z) = \psi(y)e^{k|y|/2}$, $\hat{h}(x,z) = h(x,y)e^{k|y|/2}$, and $z = \mathrm{sgn}(y)(e^{k|y|}-1)/k$. (We will be following closely the notations of ref. [3].) One of the important feature of this model is that the bound state of the higher-dimensional graviton is localized in the extra dimension. That is to say, the graviton is confined to a small region within this infinite space. The argument supporting the existence goes as follows: one considers the wave equation satisfied small gravitational fluctuations, which takes the form of a non-relativistic Schrödinger equation:

$$\left[-\frac{1}{2}\partial_z^2 + V(z)\right]\hat{\psi}(z) = m^2\hat{\psi}(z) \,. \tag{2}$$

For a simple configuration of single domain wall, one has the so-called volcano-potential.

$$V(z) = \frac{15k^2}{8(k|z|+1)^2} - \frac{3k}{2}\delta(z) \,. \tag{3}$$

(We will be working in the units of $\hbar = 1, \mathrm{mass} = M = 1$.)

First of all there is a single normalizable bound state mode, which is the graviton of the $(3+1)$-dimensional world, supported by the delta function term in the potential:

$$\hat{\psi}_0(z) = k^{-1}(k|z|+1)^{-3/2} \,. \tag{4}$$

There is also a continuum Kaluza-Klein modes, with "energy" $E = m^2/2$;

$$\hat{\psi}_m(z) \sim N_m(|z| + 1/k)^{1/2} \left[Y_2(m(|z| + 1/k)) + \frac{4k^2}{\pi m^2} J_2(m(|z| + 1/k)) \right], \qquad (5)$$

where N_m is the normalization constant. The gravitational force in our effective four dimensional world is due to the exchange of the zero mode as well as continuum Kaluza-Klein modes. These modes in the continuum without any gap potentially have some problems phenomenologically. To cure this, let us now consider a brane world that is a one-dimensional periodic lattice of these worlds, separated apart by a lattice spacing of l. We will not be concerned with an exact solution of the brane configuration, but it will be quite analogous to the solution considered in ref [8]. Consider an array of n orthogonal $n + 2$ spatial dimensional branes in $(3 + n) + 1$ dimensions, with a bulk cosmological constant Λ. The tensions of the branes are taken to be identical, all equal to σ. The action for this system is

$$S = \int_M d^{4+n}x \sqrt{g_{4+n}} \left(M_*^{n+2} \frac{R}{16\pi} + \Lambda \right) - \sum_{k=1}^{n} \int_{\mathrm{k'th\ brane}} d^{3+n}x \sqrt{g_{3+n}} \sigma \,. \qquad (6)$$

In this equation M_* is the fundamental scale of the theory. Indeed a many-brane extension of the solution of Randall and Sundrum exists [15]. In this reference the positions of the branes need not be regularly spaced. However, if we consider a network of brane worlds [13], as can be realized by considering domain wall junctions, the spacing between the different domain walls will be regular. The brane configurations will appear as a solution of the BPS equations described in ref. [13]. For this system the analytic solution does not exist. However, the solutions will be a hexagonal array, and at the junctions of the domain walls will be localized gravity. This will naturally generate a two-dimensional lattice. Since the details of the lattice shape is not of our concern, we will consider an idealized system of one-dimensional array of branes. With all these pictures, the potential for the gravitational fluctuation will be a periodic potential:

$$\mathcal{V}(z) = \sum_{n=-\infty}^{\infty} V(z - nl) \,. \qquad (7)$$

As is familiar from the condensed matter physics, we expect that the energy spectrum now develops a *band structure* with gaps between the bands. The typical width of the gap will be related to the lattice spacing. One of the key question for us would be the location of the first band of allowed Kaluza-Klein modes, which will be affecting our four-dimensional effective theory most significantly.

First of all, we assume that the domain walls are locally BPS objects. Then the fluctuation equations must be a supersymmetric quantum mechanics, so we are guaranteed a massless mode which can be interepreted as the graviton. This is certainly the case for the volcano potential. In fact we can easily check that the following is the solution with zero mass:

$$\hat{\psi}_0(z) \sim \sum_{n=-\infty}^{\infty} k^{-1}(k|z-nl|+1)^{-3/2}\,. \tag{8}$$

(We have to normalize the wave function piecewise for each period.) So around each vocano potential, there will be a localized zero mode, and the shape of the zero mode will be almost the same on each brane, when the branes are well separated.

Now let us consider the Kaluza-Klein modes, whose spectrum will depend on the detailed shape of the potential. One of the key feature of the Kaluza-Klein modes regardless of the detailed shape will be that not all the energy eigenvalues will be allowed. We will be considering the cases where the potentials are well separated, that is $l > 1/k$. We will have many problems when the branes are overlapping with each other. In that case the wave functions behave as sinusoidal functions for regions away form the center of the volcano potential. This is possible because the Bessel functions $Y_n(z)$ and $J_n(z)$ behave as sinusoidal functions for large values of z, when multiplied by $\sqrt{z}$. To analyze the spectrum, we first consider a problem of a single potential. The wave function to the left of the potential is $\hat{\psi}(z) \sim Ae^{imz} + Be^{-imz}$ for $x \ll -1/k$ and the wave function to the right will be $\hat{\psi}(z) \sim Ce^{imz} + De^{-imz}$ for $x \gg 1/k$. The relation between the coefficients will be through the transmission matrix $M(m)$ as follows:

$$\begin{pmatrix} A \\ B \end{pmatrix} = M(m)\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} F(m) & G^*(m) \\ G(m) & F^*(m) \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix}. \tag{9}$$

In the above we have used the condition for real potential $V(z)$ for the general form of the transmission matrix. We have additional condition that $G(m)$ is pure imaginary if we have even potential $V(z) = V(-z)$.

The evaluation of the components of the tranmission matrix can be done using WKB approximation and we have the following solutions [22]:

$$F(m) = \theta(m) + \frac{1}{4\theta(m)}, \qquad G(m) = i\left(\theta(m) - \frac{1}{4\theta(m)}\right). \tag{10}$$

where

$$\theta(m) = \exp\left(\int_a^b dz\sqrt{2(V(z)-E)}\right). \tag{11}$$

In the above we have $V > E = m^2/2$ for $a < z < b$. For the volcano potential, and for $m^2/2 < 15k^2/8$, we have $a = -(\sqrt{15}/2m - 1/k)$ and $b = (\sqrt{15}/2m - 1/k)$. (The delta function will not contribute, because we have $V < E$.) So for the volcano potential, we have

$$\ln(\theta(m)) = 15\Big(\log\left(\mu + \sqrt{\mu^2-1}\right) - \sqrt{1-1/\mu^2}\Big). \tag{12}$$

where $\mu = \sqrt{15/4}\,k/m > 1$. The value of θ is between one and infinity, and especially for the value of m near zero, we have $F(m) \sim \theta \sim \infty$. $F(m)$ is real throughout the values of m. This large value of $F(m)$ for small m originates from the fact that $V(z)$ is thick at the base of the volcano.

Now consider a potential which is obtained by juxtaposing $N \to \infty$ of the single potential we have considered above. Then the condition for the allowed energy is that [23]

$$X = \left| \mathrm{Re}\,(e^{\mathrm{i}m\,l} F(m)) \right| \leq 1\,. \tag{13}$$

For the case at hand, we have real values for $F(m)$, so that condition (13) is simply $|\cos(ml)F(m)| \leq 1$. The function on the left-hand side is oscillating with the enveloping amplitude determined by $F(m)$. Since for all the values $F(m) \geq 1$, we see that there are necessarily forbidden zones, because certain ranges of values of m do not satisfy the condition. Especially, for the region of m near zero, $F(m) \to \infty$, so that for small enough m the condition is not satisfied, because we have $\cos(ml) \sim 1$. The first Kaluza-Klein mode develops where $\cos(ml) \sim 0$ for the smallest value of m, since there $F(m)$ is very large. Thus we have $m \sim \pi/2l$. We see that there is a forbidden zone right above the zero mode, and this is a boon for the phenomenology for the following reasons. (If we did have a phase for $F(m)$ we might not enjoy the mass gap right above the graviton.)

(i) Since there will be a gap right above the zero mode, the lowest Kaluza-Klein mode will start at, say, m_k. For this case the physics of the effective four-dimensional theory will be mostly affected by the first band and the Newton's law of gravity will be as follows:

$$V_N(r) \sim G_N \frac{m_1 m_2}{r} \left(1 + e^{-m_k r} \left(\frac{m_k}{r} + \frac{1}{r^2} \right) \right). \tag{14}$$

We have an exponential suppression of the correction to the Newton's law. This will be a generic feature of any of the models involving a network of brane worlds or "manyfold" universe, as long as there are many of the brane worlds to have periodicity.

(ii) There will be modifications in the standard model cross sections due to the virtual exchange of graviton towers. New processes not allowed in the standard model at the tree level might appear. In the case of exchange, the amplitude is proportional to the sum over the propagators of the entire Kaluza-Klein tower and can potentially diverge. Usually it is dealt with brute force regularization [12]. In the presence of a mass gap, it will behave much better, since the lattice size of the network of braneworlds will give a natural cutoff scale.

We conclude this paper with some general remarks. First of all, we have considered models with branes in the thin limit. In principle we can model the branes from (super)gravity theories, and have smooth potentials for the fluctuations. In considering such a model, we will necessarily encounter a class of SUSY quantum mechanics with periodic superpotentials, $W(x + a) = W(x)$. SUSY quantum mechanics with periodic potentials differ from non-periodic ones that it is possible for both isospectral potentials to support zero modes, whereas in the non-periodic ones either one or neither of the pair has a zero mode [24]. Most general periodic potentials which can be analytically solved involve Jacobi's elliptic functions $\text{sn}^2(z|k)$, which in various limits become Pöschl-Teller potentials, which arose in the context of Kaluza-Klein spectrum already, or periodic potential such as Scarf potential [25], which was one of the first a soluble one dimensional energy band problem in quantum mechanics with potential $V(x) \sim \text{cosec}^2(x)$. It should be desirable to have some supergravity model which lead to such a quantum mechanics system. (Some discussions in this context can be found in refs. [20, 26].)

Second, in order to have the weak scale scale, we might have to resort to the scenario of Lykken and Randall [6]. In this scenario, one has localized graviton zero mode on the "Planck" brane and one has another brane at distance y_0 away from it such that $e^{-ky_0} = \text{TeV}/M_{pl}$. To incorporate this idea into our frame work, we will regard this set of two branes as our basic unit and imagine a periodic array of them. In terms of gravity, the "Planck" branes will be dominating the scene, and most of our analysis will be qualitatively the same. The changes in the volcano potential will be introduced, and we can do the similar WKB approximation to this new system and calculate the transmission matrix. We will again get a mass gap in the Kaluza-Klein spectrum of graviton. This is all possible as long as the lattice size is larger than the typical size of the brane pair which will be $5 - 10$ Planck lengths.

More challenging problem would be to consider the full two or higher-dimensional array of braneworlds and consider a "condensed" universe and Kaluza-Klein modes in such a universe. It might be possible to have a cosmology where we have a rather chaotic initial condition and have become condensed to a network before the nucleo-synthesis [13, 14]. If there is enough symmetry, such as hexagonal symmetry, the corresponding Kaluza-Klein spectrum can in principle be obtained and even an experiment sensitive enough to explore the configuration of the brane networks might be possible. Furthermore, compactification along the large extra dimension can affect the spectrum, just like the different electronic structure of a nanotube has from a graphite on a plane [18].

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